

DeRuyter Central School District

Mathematics

Algebra II & Trigonometry

35 Standards

Problem Solving Strand

► **Standard 1: Students will build new mathematical knowledge through problem solving.**

A2.PS.1 Use a variety of problem solving strategies to understand new mathematical content

A2.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

► **Standard 2: Students will solve problems that arise in mathematics and in other contexts.**

A2.PS.3 Observe and explain patterns to formulate generalizations and conjectures

A2.PS.4 Use multiple representations to represent and explain problem situations (e.g., verbally, numerically, algebraically, graphically)

► **Standard 3: Students will apply and adapt a variety of appropriate strategies to solve problems.**

A2.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)

A2.PS.6 Use a variety of strategies to extend solution methods to other problems

A2.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

► **Standard 4: Students will monitor and reflect on the process of mathematical problem solving.**

A2.PS.8 Determine information required to solve the problem, choose methods for obtaining the information, and define parameters for acceptable solutions

A2.PS.9 Interpret solutions within the given constraints of a problem

A2.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

Reasoning and Proof Strand

➤ **Standard 5: Students will recognize reasoning and proof as fundamental aspects of mathematics.**

A2.RP.1 Support mathematical ideas using a variety of strategies

- Prove trigonometric identities

➤ **Standard 6: Students will make and investigate mathematical conjectures.**

A2.RP.2 Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion

A2.RP.3 Evaluate conjectures and recognize when an estimate or approximation is more appropriate than an exact answer

A2.RP.4 Recognize when an approximation is more appropriate than an exact answer

➤ **Standard 7: Students will develop and evaluate mathematical arguments and proofs.**

A2.RP.5 Develop, verify, and explain an argument, using appropriate mathematical ideas and language

A2.RP.6 Construct logical arguments that verify claims or counterexamples that refute claims

A2.RP.7 Present correct mathematical arguments in a variety of forms

A2.RP.8 Evaluate written arguments for validity

➤ **Standard 8: Students will select and use various types of reasoning and methods of proof.**

A2.RP.9 Support an argument by using a systematic approach to test more than one case

A2.RP.10 Devise ways to verify results, using counterexamples and informal indirect proof

A2.RP.11 Extend specific results to more general cases

A2.RP.12 Apply inductive reasoning in making and supporting mathematical conjectures

Communication Strand

➤ **Standard 9: Students will organize and consolidate their mathematical thinking through communication.**

A2.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

A2.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, and diagrams

► **Standard 10: Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.**

A2.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form

A2.CM.4 Explain relationships among different representations of a problem

A2.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid

A2.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

► **Standard 11: Students will analyze and evaluate the mathematical thinking and strategies of others.**

A2.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students

A2.CM.8 Reflect on strategies of others in relation to one's own strategy

A2.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

► **Standard 12: Students will use the language of mathematics to express mathematical ideas precisely.**

A2.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures

A2.CM.11 Represent word problems using standard mathematical notation

A2.CM.12 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale

A2.CM.13 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

Connections Strand

►Standard 13: Students will recognize and use connections among mathematical ideas.

A2.CN.1 Understand and make connections among multiple representations of the same mathematical idea

A2.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

►Standard 14: Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

A2.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations

A2.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics

A2.CN.5 Understand how quantitative models connect to various physical models and representations

►Standard 15: Students will recognize and apply mathematics in contexts outside of mathematics.

A2.CN.6 Recognize and apply mathematics to situations in the outside world

A2.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics

A2.CN.8 Develop an appreciation for the historical development of mathematics

Representation Strand

►Standard 16: Students will create and use representations to organize, record, and communicate mathematical ideas.

A2.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts

- Utilize graphing utilities to graph data and answer questions

A2.R.2 Recognize, compare, and use an array of representational forms

A2.R.3 Use representation as a tool for exploring and understanding mathematical ideas

► **Standard 17: Students will select, apply, and translate among mathematical representations to solve problems.**

A2.R.4 Select appropriate representations to solve problem situations

A2.R.5 Investigate relationships among different representations and their impact on a given problem

► **Standard 18: Students will use representations to model and interpret physical, social, and mathematical phenomena.**

A2.R.6 Use mathematics to show and understand physical phenomena (e.g., investigate sound waves using the sine and cosine functions)

A2.R.7 Use mathematics to show and understand social phenomena (e.g., interpret the results of an opinion poll)

A2.R.8 Use mathematics to show and understand mathematical phenomena (e.g., use random number generator to simulate a coin toss)

Number Sense and Operations Strand

► *Standard 19: Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

► **Standard 20: Students will understand meanings of operations and procedures, and how they relate to one another.**

Operations

A2.N.1 Evaluate numerical expressions with negative and/or fractional exponents, without the aid of a calculator (when the answers are rational numbers)

A2.N.2 Perform arithmetic operations (addition, subtraction, multiplication, division) with expressions containing irrational numbers in radical form

A2.N.3 Perform arithmetic operations with polynomial expressions containing rational coefficients

- Solve rational equations
- Solve rational inequalities

A2.N.4 Perform arithmetic operations on irrational expressions

A2.N.5 Rationalize a denominator containing a radical expression

A2.N.6 Write square roots of negative numbers in terms of i

- Understand that we write the square root of negative one as i

A2.N.7 Simplify powers of i

A2.N.8 Determine the conjugate of a complex number

A2.N.9 Perform arithmetic operations on complex numbers and write the answer in the form $a + bi$ Note: This includes simplifying expressions with complex denominators.

- Understand the entire number system from complex number to real, to rational, to irrational to imaginary

A2.N.10 Know and apply sigma notation

►**Standard 21: Students will compute accurately and make reasonable estimates.**

Algebra Strand

►**Standard 22: Students will represent and analyze algebraically a wide variety of problem solving situations.**

Equations and Inequalities

A2.A.1 Solve absolute value equations and inequalities involving linear expressions in one variable

A2.A.2 Use the discriminant to determine the nature of the roots of a quadratic equation

- Understand how the discriminant is the only part of the quadratic needed to determine the types of roots
- Once you have determine the roots, know how that corresponds graphically

A2.A.3 Solve systems of equations involving one linear equation and one quadratic equation algebraically Note: This includes rational equations that result in linear equations with extraneous roots.

A2.A.4 Solve quadratic inequalities in one and two variables, algebraically and graphically

A2.A.5 Use direct and inverse variation to solve for unknown values

- Understand how inverse variation is related to Rectangular Hyperbolas

A2.A.6 Solve an application which results in an exponential function

►**Standard 23: Students will perform algebraic procedures accurately.**

Variables and Expressions

A2.A.7 Factor polynomial expressions completely, using any combination of the following techniques: common factor extraction, difference of two perfect squares, quadratic trinomials

A2.A.8 Apply the rules of exponents to simplify expressions involving negative and/or fractional exponents

A2.A.9 Rewrite algebraic expressions that contain negative exponents using only positive exponents

A2.A.10 Rewrite algebraic expressions with fractional exponents as radical expressions

A2.A.11 Rewrite algebraic expressions in radical form as expressions with fractional exponents

A2.A.12 Evaluate exponential expressions, including those with base e

A2.A.13 Simplify radical expressions

A2.A.14 Perform addition, subtraction, multiplication, and division of radical expressions

A2.A.15 Rationalize denominators involving algebraic radical expressions

- Understand the proper form to represent radicals

A2.A.16 Perform arithmetic operations with rational expressions and rename to lowest terms

A2.A.17 Simplify complex fractional expressions

A2.A.18 Evaluate logarithmic expressions in any base

- Memorize change of base formula
- Graph logarithmic functions using the change of base

A2.A.19 Apply the properties of logarithms to rewrite logarithmic expressions in equivalent forms

- Understand the relationship between exponential and logarithmic functions

Equations and Inequalities

A2.A.20 Determine the sum and product of the roots of a quadratic equation by examining its coefficients

A2.A.21 Determine the quadratic equation, given the sum and product of its roots

A2.A.22 Solve radical equations

A2.A.23 Solve rational equations and inequalities

A2.A.24 Know and apply the technique of completing the square

A2.A.25 Solve quadratic equations, using the quadratic formula

A2.A.26 Find the solution to polynomial equations of higher degree that can be solved using factoring and/or the quadratic formula

A2.A.27 Solve exponential equations with and without common bases

A2.A.28 Solve a logarithmic equation by rewriting as an exponential equation

- Use rules to solve any logarithmic equation

► **Standard 24: Students will recognize, use, and represent algebraically patterns, relations, and functions.**

Patterns, Relations, and Functions

A2.A.29 Identify an arithmetic or geometric sequence and find the formula for its n th term

A2.A.30 Determine the common difference in an arithmetic sequence

A2.A.31 Determine the common ratio in a geometric sequence

A2.A.32 Determine a specified term of an arithmetic or geometric sequence

A2.A.33 Specify terms of a sequence, given its recursive definition

A2.A.34 Represent the sum of a series, using sigma notation

A2.A.35 Determine the sum of the first n terms of an arithmetic or geometric series

A2.A.36 Apply the binomial theorem to expand a binomial and determine a specific term of a binomial expansion

A2.A.37 Define a relation and function

A2.A.38 Determine when a relation is a function

- Use the vertical line test to determine functions

A2.A.39 Determine the domain and range of a function from its equation

- Verify the information graphically

A2.A.40 Write functions in functional notation

A2.A.41 Use functional notation to evaluate functions for given values in the domain

- Add, subtract, multiply and divide functions

A2.A.42 Find the composition of functions

A2.A.43 Determine if a function is one-to-one, onto, or both

- Use the vertical and horizontal line test

A2.A.44 Define the inverse of a function

- Determine the inverse both algebraically and graphically

A2.A.45 Determine the inverse of a function and use composition to justify the result

A2.A.46 Perform transformations with functions and relations: $f(x+a)$, $f(x)+a$, $f(-x)$, $-f(x)$, $af(x)$

Coordinate Geometry

A2.A.47 Determine the center-radius form for the equation of a circle in standard form

- Graph a circle in a graphing calculator by solving for y

A2.A.48 Write the equation of a circle, given its center and a point on the circle

A2.A.49 Write the equation of a circle from its graph

A2.A.50 Approximate the solution to polynomial equations of higher degree by inspecting the graph

A2.A.51 Determine the domain and range of a function from its graph

A2.A.52 Identify relations and functions, using graphs

A2.A.53 Graph exponential functions of the form $y = b^x$ for positive values of b , including $b = e$

A2.A.54 Graph logarithmic functions, using the inverse of the related exponential function

Trigonometric Functions

A2.A.55 Express and apply the six trigonometric functions as ratios of the sides of a right triangle

- Understand and know how to determine the relationship between the sides of a right triangle (SOH CAH TOA)

A2.A.56 Know the exact and approximate values of the sine, cosine, and tangent of 0° , 30° , 45° , 60° , 90° , 180° , and 270° angles

- Learn how to find the values of the multiples of 30, 45, and 60
- Determine if the value is positive or negative depending upon the quadrant the terminal side lies

- A2.A.57 Sketch and use the reference angle for angles in standard position
- A2.A.58 Know and apply the co-function and reciprocal relationships between trigonometric ratios
- Memorize the cosecant, secant, and cotangent functions
- A2.A.59 Use the reciprocal and co-function relationships to find the value of the secant, cosecant, and cotangent of 0° , 30° , 45° , 60° , 90° , 180° , and 270° angles
- Learn how to find the values of the multiples of 30, 45, and 60
 - Determine if the value is positive or negative depending upon the quadrant the terminal side lies
- A2.A.60 Sketch the unit circle and represent angles in standard position
- Understand how the unit circle and trigonometry are related to the coordinate graph
- A2.A.61 Determine the length of an arc of a circle, given its radius and the measure of its central angle
- Memorize the formula $\theta = s / r$; where s is the arc length and r is the radius
- A2.A.62 Find the value of trigonometric functions, if given a point on the terminal side of angle θ
- A2.A.63 Restrict the domain of the sine, cosine, and tangent functions to ensure the existence of an inverse function
- A2.A.64 Use inverse functions to find the measure of an angle, given its sine, cosine, or tangent
- A2.A.65 Sketch the graph of the inverses of the sine, cosine, and tangent functions
- A2.A.66 Determine the trigonometric functions of any angle, using technology
- A2.A.67 Justify the Pythagorean identities
- A2.A.68 Solve trigonometric equations for all values of the variable from 0° to 360°
- Memorize the 6 basic identities
 - Solve first degree trigonometric equations
 - Use factoring to solve trigonometric equations
 - Use the quadratic formula to solve trigonometric equations
 - Use substitution to solve trigonometric equations
- A2.A.69 Determine amplitude, period, frequency, and phase shift, given the graph or equation of a periodic function
- A2.A.70 Sketch and recognize one cycle of a function of the form $y = A \sin Bx$ or $y = A \cos Bx$
- A2.A.71 Sketch and recognize the graphs of the functions $y = \sec(x)$, $y = \csc(x)$, $y = \tan(x)$, and $y = \cot(x)$

A2.A.72 Write the trigonometric function that is represented by a given periodic graph

A2.A.73 Solve for an unknown side or angle, using the Law of Sines or the Law of Cosines

- Determine when to use the Law of Sines and the Law of Cosines
- Use these formulas to solve application problems
- Use these formulas to solve vector problems

A2.A.74 Determine the area of a triangle or a parallelogram, given the measure of two sides and the included angle

A2.A.75 Determine the solution(s) from the SSA situation (ambiguous case)

- Use the Law of Sines to solve problems involving the ambiguous case

A2.A.76 Apply the angle sum and difference formulas for trigonometric functions

A2.A.77 Apply the double-angle and half-angle formulas for trigonometric functions

Geometry Strand

➤***Standard 25: Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.***

➤***Standard 26: Students will identify and justify geometric relationships, formally and informally.***

➤***Standard 27: Students will apply transformations and symmetry to analyze problem solving situations.***

➤***Standard 28: Students will apply coordinate geometry to analyze problem solving situations.***

Measurement Strand

➤**Standard 29: Students will determine what can be measured and how, using appropriate methods and formulas.**

Units of Measurement

A2.M.1 Define radian measure

A2.M.2 Convert between radian and degree measures

➤***Standard 30: Students will use units to give meaning to measurements.***

► **Standard 31: Students will understand that all measurement contains error and be able to determine its significance.**

► **Standard 32: Students will develop strategies for estimating measurements.**

Statistics and Probability Strand

Standard 33: Students will collect, organize, display, and analyze data.

Collection of Data

A2.S.1 Understand the differences among various kinds of studies (e.g., survey, observation, controlled experiment)

- Use Sigma Notation to evaluate problems

A2.S.2 Determine factors which may affect the outcome of a survey

Organization and Display of Data

A2.S.3 Calculate measures of central tendency with group frequency distributions

A2.S.4 Calculate measures of dispersion (range, quartiles, interquartile range, standard deviation, variance) for both samples and populations

A2.S.5 Know and apply the characteristics of the normal distribution

- Understand the types of data that are normally distributed
- Utilize the Normal Curve to evaluate data

► **Standard 34: Students will make predictions that are based upon data analysis.**

Predictions from Data

A2.S.6 Determine from a scatter plot whether a linear, logarithmic, exponential, or power regression model is most appropriate

A2.S.7 Determine the function for the regression model, using appropriate technology, and use the regression function to interpolate and extrapolate from the data

A2.S.8 Interpret within the linear regression model the value of the correlation coefficient as a measure of the strength of the relationship

► **Standard 35: Students will understand and apply concepts of probability.**

Probability

A2.S.9 Differentiate between situations requiring permutations and those requiring combinations

A2.S.10 Calculate the number of possible permutations (${}_n P_r$) of n items taken r at a time

A2.S.11 Calculate the number of possible combinations (${}_n C_r$) of n items taken r at a time

A2.S.12 Use permutations, combinations, and the Fundamental Principle of Counting to determine the number of elements in a sample space and a specific subset (event)

A2.S.13 Calculate theoretical probabilities, including geometric applications

A2.S.14 Calculate empirical probabilities

A2.S.15 Know and apply the binomial probability formula to events involving the terms exactly, at least, and at most

- Use Pascal's Triangle to expand binomials

A2.S.16 Use the normal distribution as an approximation for binomial probabilities