

DeRuyter Central School District

Mathematics

Integrated Geometry

35 Standards

Problem Solving Strand

► **Standard 1: Students will build new mathematical knowledge through problem solving.**

G.PS.1 Use a variety of problem solving strategies to understand new mathematical content

► **Standard 2: Students will solve problems that arise in mathematics and in other contexts.**

G.PS.2 Observe and explain patterns to formulate generalizations and conjectures

G.PS.3 Use multiple representations to represent and explain problem situations (e.g., spatial, geometric, verbal, numeric, algebraic, and graphical representations)

► **Standard 3: Students will apply and adapt a variety of appropriate strategies to solve problems.**

G.PS.4 Construct various types of reasoning, arguments, justifications and methods of proof for problems

- Use of Laws of Logic to construct Logic Proofs to reach a valid conclusion.
- Using the Given statements
- Sequencing
- Strategy / Use of common patterns

G.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)

G.PS.6 Use a variety of strategies to extend solution methods to other problems

- Comparing multiple solutions for the same problem (proof).

G.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

- Work in pairs to sequence statements + reasons.

► **Standard 4: Students will monitor and reflect on the process of mathematical problem solving.**

G.PS.8 Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions

- Practice identifying useable statements/reasons vs inappropriate statements/reasons.

G.PS.9 Interpret solutions within the given constraints of a problem

G.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

Reasoning and Proof Strand

► **Standard 5: Students will recognize reasoning and proof as fundamental aspects of mathematics.**

G.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies

G.RP.2 Recognize and verify, where appropriate, geometric relationships of perpendicularity, parallelism, congruence, and similarity, using algebraic strategies

► **Standard 6: Students will make and investigate mathematical conjectures.**

G.RP.3 Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion

► **Standard 7: Students will develop and evaluate mathematical arguments and proofs.**

G.RP.4 Provide correct mathematical arguments in response to other students' conjectures, reasoning, and arguments

G.RP.5 Present correct mathematical arguments in a variety of forms

G.RP.6 Evaluate written arguments for validity

- Logic proof.

► **Standard 8: Students will select and use various types of reasoning and methods of proof.**

G.RP.7 Construct a proof using a variety of methods (e.g., deductive, analytic, transformational)

G.RP.8 Devise ways to verify results or use counterexamples to refute incorrect statements

- Use of indirect proof !

G.RP.9 Apply inductive reasoning in making and supporting mathematical conjectures

Communication Strand

►Standard 9: Students will organize and consolidate their mathematical thinking through communication.

G.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

G.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, and diagrams

►Standard 10: Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

G.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form

- Use of appropriate format for formal proof.

G.CM.4 Explain relationships among different representations of a problem

G.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid

G.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

- Classroom discussion.

►Standard 11: Students will analyze and evaluate the mathematical thinking and strategies of others.

G.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students

G.CM.8 Reflect on strategies of others in relation to one's own strategy

- Compare multiple solutions / sequencing for common proofs.

G.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

►Standard 12: Students will use the language of mathematics to express mathematical ideas precisely.

G.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures

G.CM.11 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and geometric diagrams

G.CM.12 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

Connections Strand

►**Standard 13: Students will recognize and use connections among mathematical ideas.**

G.CN.1 Understand and make connections among multiple representations of the same mathematical idea

- Use of numerical or numerical/geometric examples (specific) to understand a general concept. (Ex. The measure of an exterior angle of a triangle...actual examples...)

G.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

►**Standard 14: Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.**

G.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations

G.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics

G.CN.5 Understand how quantitative models connect to various physical models and representations

►**Standard 15: Students will recognize and apply mathematics in contexts outside of mathematics.**

G.CN.6 Recognize and apply mathematics to situations in the outside world

G.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics

G.CN.8 Develop an appreciation for the historical development of mathematics

Representation Strand

►**Standard 16: Students will create and use representations to organize, record, and communicate mathematical ideas.**

G.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects

created using technology as representations of mathematical concepts

- Diagrams !!!!!!!!!!!!!!!
- Use of models for surface area and volume of solids.

G.R.2 Recognize, compare, and use an array of representational forms

G.R.3 Use representation as a tool for exploring and understanding mathematical ideas

► **Standard 17: Students will select, apply, and translate among mathematical representations to solve problems.**

G.R.4 Select appropriate representations to solve problem situations

- Game activities used to classify / discuss triangles and quadrilaterals.

G.R.5 Investigate relationships between different representations and their impact on a given problem

- Compare and contrast (common facts for a rhombus and a parallelogram...)

► **Standard 18: Students will use representations to model and interpret physical, social, and mathematical phenomena.**

G.R.6 Use mathematics to show and understand physical phenomena (e.g., determine the number of gallons of water in a fish tank)

G.R.7 Use mathematics to show and understand social phenomena (e.g., determine if conclusions from another person's argument have a logical foundation)

- Practice/game activity "find the errors..."

G.R.8 Use mathematics to show and understand mathematical phenomena (e.g., use investigation, discovery, conjecture, reasoning, arguments, justification and proofs to validate that the two base angles of an isosceles triangle are congruent)

Number Sense and Operations Strand

► **Standard 19: Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.**

► **Standard 20: Students will understand meanings of operations and procedures, and how they relate to one another.**

► **Standard 21: Students will compute accurately and make reasonable estimates.**

Algebra Strand

Note: The algebraic skills and concepts within the Algebra process and content performance indicators must be maintained and applied as students are asked to investigate, make conjectures, give rationale, and justify or prove geometric concepts.

► **Standard 22: Students will represent and analyze algebraically a wide variety of problem solving situations.**

► **Standard 23: Students will perform algebraic procedures accurately.**

► **Standard 24: Students will recognize, use, and represent algebraically patterns, relations, and functions.**

Geometry Strand

► **Standard 25: Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.**

Geometric Relationships

Note: Two-dimensional geometric relationships are addressed in the Informal and Formal Proofs band.

- G.G.1 Know and apply that if a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them
- G.G.2 Know and apply that through a given point there passes one and only one plane perpendicular to a given line
- G.G.3 Know and apply that through a given point there passes one and only one line perpendicular to a given plane
- G.G.4 Know and apply that two lines perpendicular to the same plane are coplanar
- G.G.5 Know and apply that two planes are perpendicular to each other if and only if one plane contains a line perpendicular to the second plane
- G.G.6 Know and apply that if a line is perpendicular to a plane, then any line perpendicular to the given line at its point of intersection with the given plane is in the given plane
- G.G.7 Know and apply that if a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane
- G.G.8 Know and apply that if a plane intersects two parallel planes, then the intersection is two parallel lines

G.G.9 Know and apply that if two planes are perpendicular to the same line, they are parallel

G.G.10 Know and apply that the lateral edges of a prism are congruent and parallel

G.G.11 Know and apply that two prisms have equal volumes if their bases have equal areas and their altitudes are equal

G.G.12 Know and apply that the volume of a prism is the product of the area of the base and the altitude

- Models / diagrams (practice 3-D sketches) ...

G.G.13 Apply the properties of a regular pyramid, including:

- lateral edges are congruent
- lateral faces are congruent isosceles triangles
- volume of a pyramid equals one-third the product of the area of the base and the altitude

G.G.14 Apply the properties of a cylinder, including:

- bases are congruent
- volume equals the product of the area of the base and the altitude
- lateral area of a right circular cylinder equals the product of an altitude and the circumference of the base

G.G.15 Apply the properties of a right circular cone, including:

- lateral area equals one-half the product of the slant height and the circumference of its base
- volume is one-third the product of the area of its base and its altitude

G.G.16 Apply the properties of a sphere, including:

- the intersection of a plane and a sphere is a circle
- a great circle is the largest circle that can be drawn on a sphere
- two planes equidistant from the center of the sphere and intersecting the sphere do so in congruent circles
- surface area is $4\pi r^2$
- volume is $\frac{4}{3}\pi r^3$
- *Constructions*

G.G.17 Construct a bisector of a given angle, using a straightedge and compass, and justify the construction

G.G.18 Construct the perpendicular bisector of a given segment, using a straightedge and compass, and justify the construction

G.G.19 Construct lines parallel (or perpendicular) to a given line through a given point, using a straightedge and compass, and justify the construction

G.G.20 Construct an equilateral triangle, using a straightedge and compass, and justify the construction

- *Locus*

G.G.21 Investigate and apply the concurrence of medians, altitudes, angle bisectors, and perpendicular bisectors of triangles

G.G.22 Solve problems using compound loci

G.G.23 Graph and solve compound loci in the coordinate plane

► **Standard 26: Students will identify and justify geometric relationships, formally and informally.**

- *Informal and Formal Proofs*

G.G.24 Determine the negation of a statement and establish its truth value

G.G.25 Know and apply the conditions under which a compound statement (conjunction, disjunction, conditional, biconditional) is true

G.G.26 Identify and write the inverse, converse, and contrapositive of a given conditional statement and note the logical equivalences

G.G.27 Write a proof arguing from a given hypothesis to a given conclusion

- Above topics specific to Logic Proof.

G.G.28 Determine the congruence of two triangles by using one of the five congruence techniques (SSS, SAS, ASA, AAS, HL), given sufficient information about the sides and/or angles of two congruent triangles

G.G.29 Identify corresponding parts of congruent triangles

G.G.30 Investigate, justify, and apply theorems about the sum of the measures of the angles of a triangle

G.G.31 Investigate, justify, and apply the isosceles triangle theorem and its converse

G.G.32 Investigate, justify, and apply theorems about geometric inequalities, using the exterior angle theorem

- G.G.33 Investigate, justify, and apply the triangle inequality theorem
- G.G.34 Determine either the longest side of a triangle given the three angle measures or the largest angle given the lengths of three sides of a triangle
- G.G.35 Determine if two lines cut by a transversal are parallel, based on the measure of given pairs of angles formed by the transversal and the lines
- G.G.36 Investigate, justify, and apply theorems about the sum of the measures of the interior and exterior angles of polygons
- G.G.37 Investigate, justify, and apply theorems about each interior and exterior angle measure of regular polygons
- G.G.38 Investigate, justify, and apply theorems about parallelograms involving their angles, sides, and diagonals
- G.G.39 Investigate, justify, and apply theorems about special parallelograms (rectangles, rhombuses, squares) involving their angles, sides, and diagonals
- G.G.40 Investigate, justify, and apply theorems about trapezoids (including isosceles trapezoids) involving their angles, sides, medians, and diagonals
- G.G.41 Justify that some quadrilaterals are parallelograms, rhombuses, rectangles, squares, or trapezoids
- G.G.42 Investigate, justify, and apply theorems about geometric relationships, based on the properties of the line segment joining the midpoints of two sides of the triangle
- G.G.43 Investigate, justify, and apply theorems about the centroid of a triangle, dividing each median into segments whose lengths are in the ratio 2:1
- G.G.44 Establish similarity of triangles, using the following theorems: AA, SAS, and SSS
- G.G.45 Investigate, justify, and apply theorems about similar triangles
- G.G.46 Investigate, justify, and apply theorems about proportional relationships among the segments of the sides of the triangle, given one or more lines parallel to one side of a triangle and intersecting the other two sides of the triangle
- G.G.47 Investigate, justify, and apply theorems about mean proportionality:
- the altitude to the hypotenuse of a right triangle is the mean proportional between the two segments along the hypotenuse
 - Geometric Mean...

- the altitude to the hypotenuse of a right triangle divides the hypotenuse so that either leg of the right triangle is the mean proportional between the hypotenuse and segment of the hypotenuse adjacent to that leg

G.G.48 Investigate, justify, and apply the Pythagorean theorem and its converse

G.G.49 Investigate, justify, and apply theorems regarding chords of a circle:

- perpendicular bisectors of chords
- the relative lengths of chords as compared to their distance from the center of the circle

G.G.50 Investigate, justify, and apply theorems about tangent lines to a circle:

- a perpendicular to the tangent at the point of tangency
- two tangents to a circle from the same external point
- common tangents of two non-intersecting or tangent circles

G.G.51 Investigate, justify, and apply theorems about the arcs determined by the rays of angles formed by two lines intersecting a circle when the vertex is:

- inside the circle (two chords)
- on the circle (tangent and chord)
- outside the circle (two tangents, two secants, or tangent and secant)

G.G.52 Investigate, justify, and apply theorems about arcs of a circle cut by two parallel lines

G.G.53 Investigate, justify, and apply theorems regarding segments intersected by a circle:

- along two tangents from the same external point
- along two secants from the same external point
- along a tangent and a secant from the same external point
- along two intersecting chords of a given circle

► **Standard 27: Students will apply transformations and symmetry to analyze problem solving situations.**

- *Transformational Geometry*
- Use Coord. Graphing for most T.G. situations/problems.

G.G.54 Define, investigate, justify, and apply isometries in the plane (rotations, reflections, translations, glide reflections) *Note: Use proper function notation.*

G.G.55 Investigate, justify, and apply the properties that remain invariant under translations, rotations, reflections, and glide reflections

G.G.56 Identify specific isometries by observing orientation, numbers of invariant points, and/or parallelism

G.G.57 Justify geometric relationships (perpendicularity, parallelism, congruence) using transformational techniques (translations, rotations, reflections)

G.G.58 Define, investigate, justify, and apply similarities (dilations and the composition of dilations and isometries)

G.G.59 Investigate, justify, and apply the properties that remain invariant under similarities

G.G.60 Identify specific similarities by observing orientation, numbers of invariant points, and/or parallelism

G.G.61 Investigate, justify, and apply the analytical representations for translations, rotations about the origin of 90° and 180° , reflections over the lines $x = 0$, $y = 0$, and $y = x$, and dilations centered at the origin

►Standard 28: Students will apply coordinate geometry to analyze problem solving situations.

Coordinate Geometry

○ *Explanations must be written in complete sentences!*

G.G.62 Find the slope of a perpendicular line, given the equation of a line

G.G.63 Determine whether two lines are parallel, perpendicular, or neither, given their equations

G.G.64 Find the equation of a line, given a point on the line and the equation of a line perpendicular to the given line

G.G.65 Find the equation of a line, given a point on the line and the equation of a line parallel to the desired line

G.G.66 Find the midpoint of a line segment, given its endpoints

G.G.67 Find the length of a line segment, given its endpoints

G.G.68 Find the equation of a line that is the perpendicular bisector of a line segment, given the endpoints of the line segment

○ *Use of mdpt.+ perp. Slopes....*

G.G.69 Investigate, justify, and apply the properties of triangles and quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas

G.G.70 Solve systems of equations involving one linear equation and one quadratic equation graphically

G.G.71 Write the equation of a circle, given its center and radius or given the endpoints of a diameter

G.G.72 Write the equation of a circle, given its graph *Note: The center is an ordered pair of integers and the radius is an integer.*

G.G.73 Find the center and radius of a circle, given the equation of the circle in center-radius form

G.G.74 Graph circles of the form $(x-h)^2 + (j-k)^2 = r^2$

Measurement Strand

- **Standard 29: Students will determine what can be measured and how, using appropriate methods and formulas.**
- **Standard 30: Students will use units to give meaning to measurements.**
- **Standard 31: Students will understand that all measurement contains error and be able to determine its significance.**
- **Standard 32: Students will develop strategies for estimating measurements.**

Statistics and Probability Strand

- **Standard 33: Students will collect, organize, display, and analyze data.**
- **Standard 34: Students will make predictions that are based upon data analysis.**
- **Standard 35: Students will understand and apply concepts of probability.**