

DeRuyter Central School District

Mathematics

Grade 6

125 Benchmarks

Problem Solving Strand

►Standard 1: Students will build new mathematical knowledge through problem solving.

6.PS.1 Know the difference between relevant and irrelevant information when solving problems

6.PS.2 Understand that some ways of representing a problem are more efficient than others

6.PS.3 Interpret information correctly, identify the problem, and generate possible strategies and solutions

►Standard 2: Students will solve problems that arise in mathematics and in other contexts.

6.PS.4 Act out or model with manipulatives activities involving mathematical content from literature

6.PS.5 Formulate problems and solutions from everyday situations

6.PS.6 Translate from a picture/diagram to a numeric expression

6.PS.7 Represent problem situations verbally, numerically, algebraically, and/or graphically

6.PS.8 Select an appropriate representation of a problem

6.PS.9 Understand the basic language of logic in mathematical situations (and, or, and not)

►Standard 3: Students will apply and adapt a variety of appropriate strategies to solve problems.

6.PS.10 Work in collaboration with others to solve problems

6.PS.11 Translate from a picture/diagram to a number or symbolic expression

6.PS.12 Use trial and error and the process of elimination to solve problems

6.PS.13 Model problems with pictures/diagrams or physical objects

6.PS.14 Analyze problems by observing patterns

- General Patterns
- Special Cases

6.PS.15 Make organized lists or charts to solve numerical problems

►Standard 4: Students will monitor and reflect on the process of mathematical problem solving.

6.PS.16 Discuss with peers to understand a problem situation

6.PS.17 Determine what information is needed to solve problem

6.PS.18 Determine the efficiency of different representations of a problem

6.PS.19 Differentiate between valid and invalid approaches

6.PS.20 Understand valid counterexamples

6.PS.21 Explain the methods and reasoning behind the problem solving strategies used

6.PS.22 Discuss whether a solution is reasonable in the context of the original problem

6.PS.23 Verify results of a problem

Reasoning and Proof Strand

►Standard 5: Students will recognize reasoning and proof as fundamental aspects of mathematics.

6.RP.1 Recognize that mathematical ideas can be supported using a variety of strategies

6.RP.2 Understand that mathematical statements can be supported, using models, facts, and relationships to explain their thinking

►Standard 6: Students will make and investigate mathematical conjectures.

6.RP.3 Investigate conjectures, using arguments and appropriate mathematical terms

6.RP.4 Make and evaluate conjectures, using a variety of strategies

►Standard 7: Students will develop and evaluate mathematical arguments and proofs.

6.RP.5 Justify general claims or conjectures, using manipulatives, models, expressions, and mathematical relationships

6.RP.6 Develop and explain an argument verbally, numerically, algebraically, and/or graphically

6.RP.7 Verify claims other students make, using examples and counterexamples when appropriate

►Standard 8: Students will select and use various types of reasoning and methods of proof.

6.RP.8 Support an argument through examples/counterexamples and special cases

6.RP.9 Devise ways to verify results

Communication Strand

►Standard 9: Students will organize and consolidate their mathematical thinking through communication.

6.CM.1 Provide an organized thought process that is correct, complete, coherent, and clear

6.CM.2 Explain a rationale for strategy selection

6.CM.3 Organize and accurately label work

►Standard 10: Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

6.CM.4 Share organized mathematical ideas through the manipulation of objects, numerical tables, drawings, pictures, charts, graphs, tables, diagrams, models, and symbols in written and verbal form

6.CM.5 Answer clarifying questions from others

►Standard 11: Students will analyze and evaluate the mathematical thinking and strategies of others.

6.CM.6 Understand mathematical solutions shared by other students

6.CM.7 Raise questions that elicit, extend, or challenge others' thinking

6.CM.8 Consider strategies used and solutions found by others in relation to their own work

►**Standard 12: Students will use the language of mathematics to express mathematical ideas precisely.**

6.CM.9 Increase their use of mathematical vocabulary and language when communicating with others

6.CM.10 Use appropriate vocabulary when describing objects, relationships, mathematical solutions, and rationale

6.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

Connections Strand

►**Standard 13: Students will recognize and use connections among mathematical ideas.**

6.CN.1 Understand and make connections and conjectures in their everyday experiences to mathematical ideas

6.CN.2 Explore and explain the relationship between mathematical ideas

6.CN.3 Connect and apply mathematical information to solve problems

►**Standard 14: Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.**

6.CN.4 Understand multiple representations and how they are related

6.CN.5 Model situations with objects and representations and be able to draw conclusions

►**Standard 15: Students will recognize and apply mathematics in contexts outside of mathematics.**

6.CN.6 Recognize and provide examples of the presence of mathematics in their daily lives

6.CN.7 Apply mathematics to problem situations that develop outside of mathematics

6.CN.8 Investigate the presence of mathematics in careers and areas of interest

6.CN.9 Recognize and apply mathematics to other disciplines and areas of interest

Representation Strand

► **Standard 16: Students will create and use representations to organize, record, and communicate mathematical ideas.**

6.R.1 Use physical objects, drawings, charts, tables, graphs, symbols, equations, or objects created using technology as representations

6.R.2 Explain, describe, and defend mathematical ideas using representations

6.R.3 Read, interpret, and extend external models

6.R.4 Use standard and nonstandard representations with accuracy and detail

► **Standard 17: Students will select, apply, and translate among mathematical representations to solve problems.**

6.R.5 Use representations to explore problem situations

6.R.6 Investigate relationships between different representations and their impact on a given problem

► **Standard 18: Students will use representations to model and interpret physical, social, and mathematical phenomena.**

6.R.7 Use mathematics to show and understand physical phenomena (e.g., determine the perimeter of a bulletin board)

6.R.8 Use mathematics to show and understand social phenomena (e.g., construct tables to organize data showing book sales)

6.R.9 Use mathematics to show and understand mathematical phenomena (e.g., Find the missing value: $(3 + 4) + 5 = 3 + (4 + \underline{\quad})$)

Number Sense and Operations Strand

► **Standard 19: Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.**

Number Systems

6.N.1 Read and write whole numbers to trillions

- know place value from ones to trillions

- standard notation
 - number-and-word notation
- 6.N.2 Define and identify the commutative and associative properties of addition and multiplication
- Commutative: $a + b = b + a$ $a \times b = b \times a$
 - Associative: $a + (b + c) = (a + b) + c$ $a \times (b \times c) = (a \times b) \times c$
- 6.N.3 Define and identify the distributive property of multiplication over addition
- 6.N.4 Define and identify the identity and inverse properties of addition and multiplication
- identity property: $a + 0 = a$ $a \times 1 = a$
 - inverse property: $a + (-a) = 0$ $a \times \frac{1}{a} = 1$
- 6.N.5 Define and identify the zero property of multiplication
- $a \times 0 = 0$
- 6.N.6 Understand the concept of rate
- rate – a ratio of two measurements having different units
- 6.N.7 Express equivalent ratios as a proportion
- proportions are made up of two equivalent fractions
- 6.N.8 Distinguish the difference between rate and ratio
- a ratio is a comparison of two numbers by division
 - a rate is a ratio with two different units
- 6.N.9 Solve proportions using equivalent fractions
- use cross-products to set up equations
- 6.N.10 Verify the proportionality using the product of the means equals the product of the extremes
- cross-products
- 6.N.11 Read, write, and identify percents of a whole (0% to 100%)
- 6.N.12 Solve percent problems involving percent, rate, and base
- 6.N.13 Define absolute value and determine the absolute value of rational numbers (including positive and negative)
- distance that a number is from zero on a number line
 - absolute value is always a positive number because distance cannot be negative
- 6.N.14 Locate rational numbers on a number line (including positive and negative)

- rational number – any number that can be written as a fraction
- integers, fractions, terminating, repeating decimals

6.N.15 Order rational numbers (including positive and negative)

► Standard 20: Students will understand meanings of operations and procedures, and how they relate to one another.

Operations

6.N.16 Add and subtract fractions with unlike denominators

- Addition/Subtraction – must make equivalent fractions with common denominators before adding/subtracting
- Least Common Multiple (Least Common Denominator)
- Simplest Form – Reduce (Greatest Common Factor)

6.N.17 Multiply and divide fractions with unlike denominators

- Multiplication – multiply numerators and then denominators
- Division – multiply by the reciprocal
- Cross-Cancel (Greatest Common Factor)
- Simplest Form – Reduce (Greatest Common Factor)

6.N.18 Add, subtract, multiply, and divide mixed numbers with unlike denominators

- Addition/Subtraction – must rename fractions with a common denominator before adding/subtracting
- Multiplication – change mixed numbers to improper fractions before multiplying
- Division – change mixed numbers to improper fractions before multiplying by the reciprocal
- Simplest Form – Reduce (Greatest Common Factor)

6.N.19 Identify the multiplicative inverse (reciprocal) of a number

- the product of a number and its multiplicative inverse is always 1

6.N.20 Represent fractions as terminating or repeating decimals

- fraction to decimal conversions (divide fraction to get decimal)
- terminating decimal – a decimal whose digits end
- repeating decimal – a decimal whose digits repeat in groups of one or more

6.N.21 Find multiple representations of rational numbers (fractions, decimals, and percents 0 to 100)

- fraction, decimal, percent conversions

6.N.22 Evaluate numerical expressions using order of operations (may include exponents of two and three)

- PEMDAS
 - Parentheses
 - Exponents
 - Multiplication
 - Division
 - Addition
 - Subtraction

6.N.23 Represent repeated multiplication in exponential form

- Exponential Form (Notation) – a way of representing repeated multiplication by the same factor
- Base – the number that is being multiplied (factor)
- Exponent – indicates how many times the base is used as a factor
- Powers of 10 (a number in exponential form with a base of 10)
- Powers of other whole numbers
- Scientific Notation – a number written as the product of a power of ten and a number that is at least 1 but less than 10

6.N.24 Represent exponential form as repeated multiplication

- Powers of 10
- Powers of other whole numbers

6.N.25 Evaluate expressions having exponents where the power is an exponent of one, two, or three

►Standard 21: Students will compute accurately and make reasonable estimates.

Estimation

6.N.26 Estimate a percent of quantity (0% to 100%)

6.N.27 Justify the reasonableness of answers using estimation (including rounding)

Algebra Strand

►Standard 22: Students will represent and analyze algebraically a wide variety of problem solving situations.

Variables and Expressions

6.A.1 Translate two-step verbal expressions into algebraic expressions

- Variable – a letter or other symbol that represents a number

►Standard 23: Students will perform algebraic procedures accurately.

Variables and Expressions

6.A.2 Use substitution to evaluate algebraic expressions (may include exponents of one, two and three)

- substitute values for variables and evaluate

Equations and Inequalities

6.A.3 Translate two-step verbal sentences into algebraic equations

- know math operations terminology
 - sum, total, more than, all together, greater than, increased by
 - difference, minus, less than, less, decreased by
 - product, multiplied by, times, twice, of
 - quotient, divided by, each, per, rate

6.A.4 Solve and explain two-step equations involving whole numbers using inverse operations

6.A.5 Solve simple proportions within context

6.A.6 Evaluate formulas for given input values (circumference, perimeter, area, volume, distance, temperature, interest, etc.)

- Area – the amount of space inside a closed figure
 - squares, rectangles, triangles
- Perimeter – the distance around a closed figure
- Circumference –
- Volume -

►Standard 24: Students will recognize, use, and represent algebraically patterns, relations, and functions.

Geometry Strand

►Standard 25: Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.

Shapes

6.G.1 Calculate the length of corresponding sides of similar triangles, using proportional reasoning

6.G.2 Determine the area of triangles and quadrilaterals (squares and rectangles) and develop formulas

6.G.3 Use a variety of strategies to find the area of regular and irregular polygons

- 6.G.4 Determine the volume of rectangular prisms by counting cubes and develop the formula
- 6.G.5 Identify radius, diameter, chords, arc, and sector of a circle
- circle – set of all points in a plane that are the same distance from a given point called the center
 - radius – distance from center point of circle to the outside of circle
 - diameter – distance across a circle through the center point
 - chord – a line segment joining two points on a circle
 - arc – part of a circle (semicircle) from one point on the circle to another
 - sector – a region bound by an arc and two radii of a circle (resembles a slice of pizza)
- 6.G.6 Understand the relationship between the diameter and radius of a circle
- 6.G.7 Determine the area and circumference of a circle, using the appropriate formula
- 6.G.8 Calculate the area of a sector of a circle, given the measure of a central angle and the radius of the circle
- 6.G.9 Understand the relationship between the circumference and the diameter of a circle

► **Standard 26: Students will identify and justify geometric relationships, formally and informally.**

► **Standard 27: Students will apply transformations and symmetry to analyze problem solving situations.**

► **Standard 28: Students will apply coordinate geometry to analyze problem solving situations.**

Coordinate Geometry

- 6.G.10 Identify and plot points in all four quadrants
- origin
 - ordered number pair
 - x and y-axis
 - coordinate
 - midpoint
- 6.G.11 Calculate the area of basic polygons drawn on a coordinate plane (rectangles and shapes composed of rectangles having sides with integer lengths)

Measurement Strand

► **Standard 29: Students will determine what can be measured and how, using appropriate methods and formulas.**

Units of Measurement

- 6.M.1 Measure capacity and calculate volume of a rectangular prism
- 6.M.2 Identify customary units of capacity (cups, pints, quarts, and gallons)
- 6.M.3 Identify equivalent customary units of capacity (cups to pints, pints to quarts, and quarts to gallons)
- 2 cups = 1 pint
 - 2 pints = 1 quart
 - 4 quarts = 1 gallon
- 6.M.4 Identify metric units of capacity (liter and milliliter)
- 6.M.5 Identify equivalent metric units of capacity (milliliter to liter and liter to milliliter)
- 1000 milliliters = 1 liter
 - 1 liter = 1000 milliliters

Tools and Methods

- 6.M.6 Determine the tool and technique to measure with an appropriate level of precision: capacity

► **Standard 30: Students will use units to give meaning to measurements.**

► **Standard 31: Students will understand that all measurement contains error and be able to determine its significance.**

► **Standard 32: Students will develop strategies for estimating measurements.**

Estimation

- 6.M.7 Estimate volume, area, and circumference (see figures identified in geometry strand)
- 6.M.8 Justify the reasonableness of estimates
- 6.M.9 Determine personal references for capacity

Statistics and Probability Strand

►Standard 33: Students will collect, organize, display, and analyze data.

Collection of Data

- 6.S.1 Develop the concept of sampling when collecting data from a population and decide the best method to collect data for a particular question
- sample – a randomly selected group chosen for the purpose of collecting data
 - population – the entire group of items or individuals from which the samples under consideration are taken

Organization and Display of Data

- 6.S.2 Record data in a frequency table
- Frequency Table – tally of a set of data
- 6.S.3 Construct Venn diagrams to sort data
- Venn Diagram – a picture that uses circles to show relationships among sets
 - Shared data is placed in the intersection of the two circles.
- 6.S.4 Determine and justify the most appropriate graph to display a given set of data (pictograph, bar graph, line graph, histogram, circle graph, line plot, step graph)

Analysis of Data

- 6.S.5 Determine the mean, mode and median for a given set of data
- mean – the average of a set of data (the sum of the data divided by the number of items in the data set)
 - median – the middle number in a set of data arranged from least to greatest
 - mode – the number(s) that occur most in a data set
- 6.S.6 Determine the range for a given set of data
- maximum – the largest number in a data set
 - minimum – the smallest number in a data set
 - range - the difference between the greatest and least numbers in a data set (maximum – minimum)
- 6.S.7 Read and interpret graphs
- bar graphs
 - line graphs
 - circle graphs
 - step graphs
 - line plots
 - histograms

► **Standard 34: Students will make predictions that are based upon data analysis.**

Predictions from Data

6.S.8 Justify predictions made from data

► **Standard 35: Students will understand and apply concepts of probability.**

Probability

6.S.9 List possible outcomes for compound events

- outcome – one possible result of a probability event
- compound event – an event that consists of two or more simple events
- tree diagram

6.S.10 Determine the probability of dependent events

- probability – the chance that an event will happen (ratio of the number of ways a certain event can occur to the number of possible outcomes)
- dependent events – two or more events in which the outcome of one event does affect the outcome of the other event or events

6.S.11 Determine the number of possible outcomes for a compound event by using the fundamental counting principle and use this to determine the probabilities of events when the outcomes have equal probability

- fundamental counting principle – uses multiplication of the number of ways each event in an experiment can occur to find the number of possible outcomes